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Teevan, Charles L.; Charest, Phillip G.

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A PRELIMINARY STUDY IN THE APPLICATION OF HOT-WIRE ANEMOMETRY TO THE STUDY OF REPEATED SHOCK WAVES

CHARLES L. TEEVAN
and
PHILIP G. CHAREST

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UNITED STATES
NAVAL POSTGRADUATE SCHOOL

THESIS

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IN THE APPLICATION OF HOT-WIRE ANEMOMETRY

TO THE STUDY OF REPEATED SHOCK WAVES

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Charles L. Teevan

Commander, United States Navy

and

Philip G. Charest

Lieutenant, United States Navy

Submitted in partial fulfillment of
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IN
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Abstract.

The feasibility of using a hot-wire anemometer as a means of examining the complex flow field associated with repeated plane shock waves in a shock tube is investigated. The limitations inherent in this study and the variables that must be accounted for are considered. A criterion is established for the determination of the presence of significant acoustical streaming. A method for measuring peak alternating velocities is postulated. Experimental results show good agreement with theory except for the alternating boundary layer which is apparently <u>not</u> an example of simple shear flow.

The writers wish to express their appreciation for the assistance and encouragement given them by Professor H. Medwin of the U. S. Naval Postgraduate School in this investigation.

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1. Introduction.

Investigations into the attenuation of repeated plane shock waves confined in tubes have revealed a large amount of data. In general, this experimental data has failed to agree with any existing theory on this subject. To date, all known investigations have utilized pressure sensitive instrumentation which has shown a progressive decrease in shock intensity as the shock wave traverses down the tube. This attenuation appears to be sensitive to both frequency and tube radius. Since there exists no satisfactory theory to substantiate quantitatively this observed attenuation and as the probability of obtaining more revealing data from pressure measurements seems slight, this experiment was conceived. The main purpose of which is to investigate the feasibility of applying hot-wire anemometry to determine the nature of particle velocity, near the wall of the tube, during the passage of the shock wave.

It is anticipated that such an approach will eventually reveal acoustic streaming if it exists, turbulence, the nature of the boundary layer near the wall, and within limited accuracy, the actual velocities. Information gained by this technique will extend present knowledge of repeated finite plane shock waves and when combined with prior knowledge may well lead to a workable theory that will permit the study of infinite repeated

shock waves to be conducted under the controlled laboratory conditions that a shock tube provides.

2. Characteristics of Repeated Shock Waves.

Unlike single shock waves (as generated by explosions, nuclear blasts, etc.) which propagate with greater than acoustic velocity, repeated shock waves travel with approximately the normal speed of sound. The repeated shock wave has a sawtooth form and is stable except for a gradual attenuation in amplitude as it progresses further from its source.

The sawtooth form of the shock wave is produced by distortion of a large amplitude sound wave. If this wave has sufficient amplitude, it will develop into a shock wave regardless of its initial form. This is due to the fact that the crest of the sound wave travels at a greater velocity than the trough. The initial wave form progressively distorts finally becoming a sawtooth wave at which point the shock front is fully developed. This progress is shown in Fig. 1 for an initial sine wave form. The distortion of a high amplitude sound wave into a shock wave can easily be observed using a pressure sensitive probe or microphone and an oscilloscope to detect the sound wave as it is propagated down the tube. At the beginning of the tube the normal wave form is observed; further along the tube the sound wave form becomes distorted in the direction of a sawtooth form, finally, at some further portion of the tube the shock front is fully developed and a sawtooth form is observed.

The primary characteristic of the shock wave form is its stability. It will not change form if it is a true shock and the only change it shows is a decrease in amplitude due to attenuation.

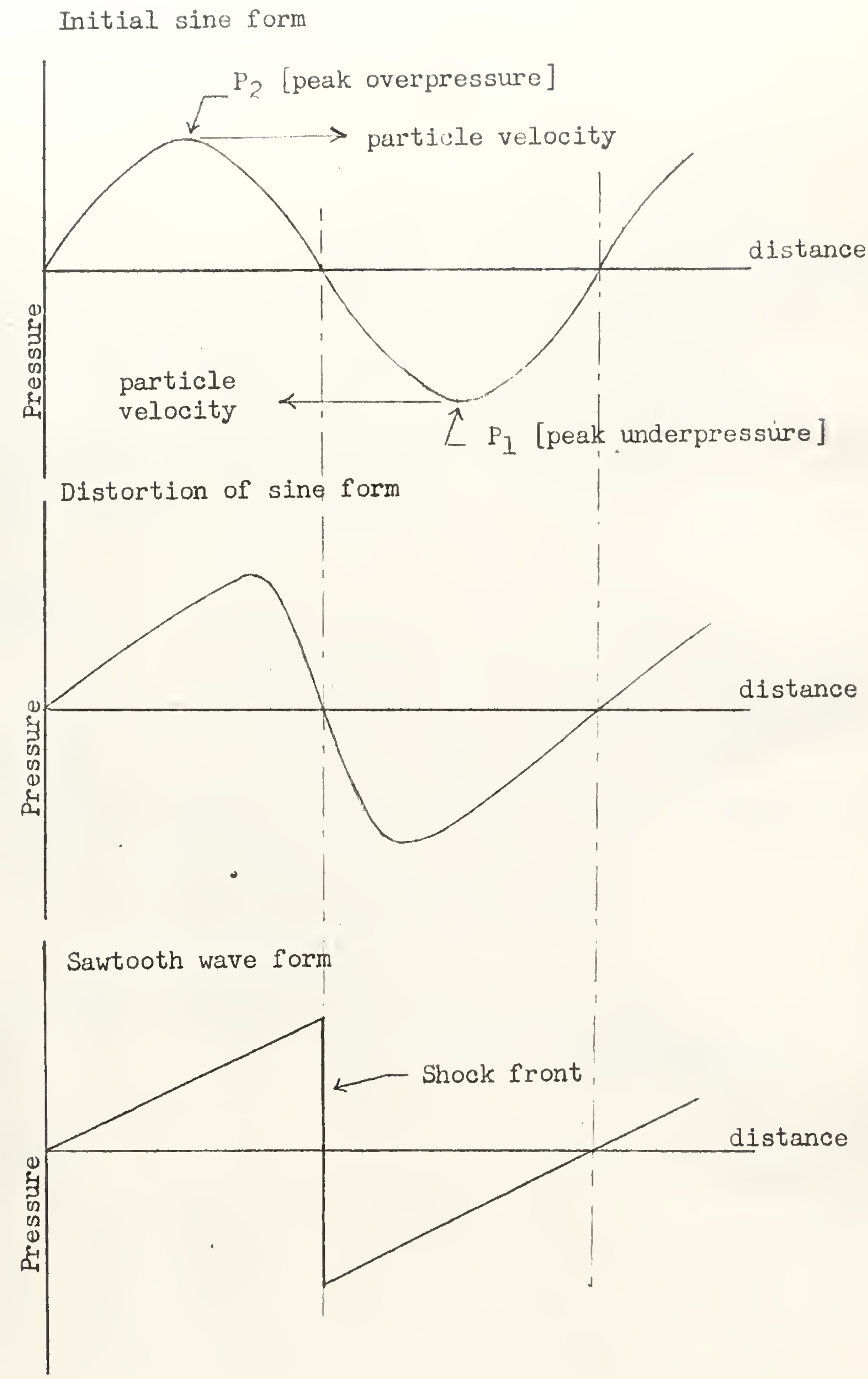


Figure 1. Developement of Sawtooth Wave

3. Theory of Hot-wire Anemometry.

The initial theory for the use of hot-wire anemometry for the determination of particle velocity was performed by King (1). The response equations (2,7) have been developed from initial theory and are based on the conservation of energy. The difference between the rate of heat input to the wire and the rate of heat loss of the wire to the moving fluid can be equated to the rate of change of stored thermal energy in the wire.

$$W - H = C(dT_W/dt) \qquad (1)$$

where:

W = Heat input to wire per unit time

H = Heat loss of wire per unit time

C = Heat capacitance of wire

If R_W is the wire resistance at a temperature T_W in ^oC and "i" is the heating current, then:

$$W = i^2 R_W \qquad (2)$$

King (1) expressed the heating loss as:

$$H = L(T_W - T_a)(k + \sqrt{2\pi k d c_p \rho u}\,) \qquad (3)$$

where:

L = length of wire

T_W = temperature of wire in ^oC

T_a = temperature of air in ^oC

k = thermal conductivity of wire

c_p = specific heat of fluid

ρ = density of fluid

d = diameter of wire

u = velocity of fluid

By defining:

$$A = kL$$

$$B' = L \sqrt{2\pi k d c_p}$$

King's expression for the heating loss becomes:

$$H = (T_W - T_a)(A + B'\sqrt{\rho u}) \qquad (4)$$

A simplification for the study of fluid motion where equilibrium can be assumed allows $C(dT_W/dt)$ to equal zero.

Therefore:

$$W - H = 0$$

or $$i^2 R_W = (T_W - T_a)(A + B'\sqrt{\rho u}) \qquad (5)$$

Unfortunately however, due to the rapidity of the variations in temperature, pressure, and fluid velocity under repeated plane shock waves equilibrium conditions do not occur. Therefore King's expression must remain as:

$$i^2 R_W - (T_W - T_a)(A + B'\sqrt{\rho u}) = C(dT_W/dt) \qquad (6)$$

The major difficulty that exists in applying hot-wire anemometry to this study is the inherent lag between the response of the wire to a velocity change and the velocity change. In previous applications of hot-wire anemometers [1,2,6,9], proper electronic compensation was introduced into the amplifier. The usage of this method has proven very satisfactory in turbulent flow fields where the variations in flow were minor as compared to the steady flow field. A more recent determination [7] of wire time

constant for large transients from steady state conditions provides a means of studying single shock phenomena (2). However during repeated shock waves steady flow conditions are non-existent.

This thesis describes a method of determining a qualitative description of the flow field in the tube, which may be extended to quantitative knowledge where applicable.

4. Description of Equipment.

The equipment used to produce the high intensity sound consists of a 15 horsepower electric motor driving an air compressor, capable of delivering 400 cubic feet of air per minute at pressures up to approximately 4 psi.* (7 psi available for short periods) This flow of air is cut by an electrically driven rotary chopper producing alternate rarefactions and condensations in an adjustable standing wave chamber. An exhaust pipe fitted to the standing wave chamber, in combination with the plugged downstream end of the propagation tube, eliminates the DC air flow.

Fig. 2 is a schematic diagram of the entire experimental set-up.

The propagation tube is a 30 foot section of rectangular seamless steel tubing with a wall thickness of approximately one-eighth inch and with inside dimensions of 1-1/4 by 2-3/4 inches. The downstream end of the tube is plugged. A termination consisting of the last eight feet of tubing was filled with a fiber-glass, wedge-shaped absorber to reduce reflections. A test for the existence of standing waves was conducted by pressure probing every inch of a two foot section of tube immediately upstream from the termination section and every foot thereafter, with an Altec 21-BR-200 microphone. Pressure readings from each station when plotted against distance showed

[illegible] attenuation with distance and the existence of standing waves of infinitesimal magnitude.[1]

Nine feet from the upstream end of the tube a cut was made for the installation of plexiglass sections. These plexiglass sections were open-ended boxes, constructed of one half inch thick plexiglass, machined to provide inside dimensions identical to those of the propagation tube. Initially the idea of installing a separate section was to provide a means of isolating a small section of the propagation tube. This was accomplished by installing diaphragms at both ends of the plexiglass section in order to eliminate the acoustic streaming in that section of the pipe, yet still allow for passage of the shock wave. Thus, inside this section, the AC velocity components would be directly available to us. Plexiglass was selected as the material for the construction of these sections because its transparent qualities afforded excellent additional opportunities for direct measurement of the position of the hot-wire probe in relation to the surface of the tube during boundary layer investigations, and the possibility of qualitatively determining the nature of the flow using smoke particles as tracers.

The hot-wire anemometer used was a Shapiro & Edwards Company, Pasadena, California, constant current model, consisting of the following units:

a. Hot-Wire Anemometer Amplifier, Model A-50B

b. Current Control Panel, Model C-50

c. Resistance Bridge, Model B-50

d. Potentiometer, Model P-50A

e. Mean Square Output Meter, Model M-50

f. Square Wave Generator, Model G-50

g. Power Supply for Amplifier Model A-50B

In conjunction with the above, Flow Corporation Probes employing platinum filaments of approximately 0.05 inches in length and 0.0005 inches in diameter were used.

Although all of the above hot-wire anemometer components were not utilized in the actual measurements, all are necessary for the setting up and checking out of the anemometer, and shall thus be described below.

The Hot-Wire Anemometer Amplifier is a low noise, high gain, wide band amplifier. Provisions are made for checking frequency response and amplification using the associated square wave generator. Maximum uncompensated gain is 5,000, when fully compensated, 2,500,000. An input selector allows selection of input from hot-wire or squarewave generator.

The Current Control Panel permitted adjustment of wire current over the range of 1 to 300 ma. An incorporated meter with full scale ranges of 10, 100, and 1000 ma provides initial adjustment of the hot-wire current.

The Potentiometer enabled the voltage or the current of the hot-wire to be accurately measured. Voltage ranges

are 0.1, 1.0, and 10 volts full scale. Current ranges are 0.01, 0.1, and 1.0 amps full scale.

The Resistance Bridge has a range of 0 to 120 ohms and may be used to measure wire resistance while the amplifier is in use without introducing noise or hum. Sensitivity is sufficient to enable a balance with only 1 ma of current.

The Mean Square Output Meter is a sensitive millivoltmeter incorporating a mirror scale for reading the thermocouple output of the amplifier.

The Square Wave Generator is a self-contained, battery operated, transistor type. The frequency is variable and the signal can be used to check frequency response to the amplifier. RMS output meter and precision attenuation enable accurate gain measurement. Time constant determination of a given wire may be determined utilizing a series of switches and attenuators to superimpose a square mean component of adjustable amplitude on mean wire current.

A Barium Titanate transducer was mounted upstream from the hot-wire probe to verify the existence of a shock wave in the propagation tube and to monitor and to determine the repetition rate. The output of the transducer was amplified by a H.H. Scott Decade Amplifier and displayed on a Tektronix 545 dual trace oscilloscope. This signal was also sent to a Hewlett Packard 400-D Vacuum Tube

Voltmeter or by way of an SKL Model 302 Variable Electronic Filter for isolation of the fundamental frequency or a desired harmonic.

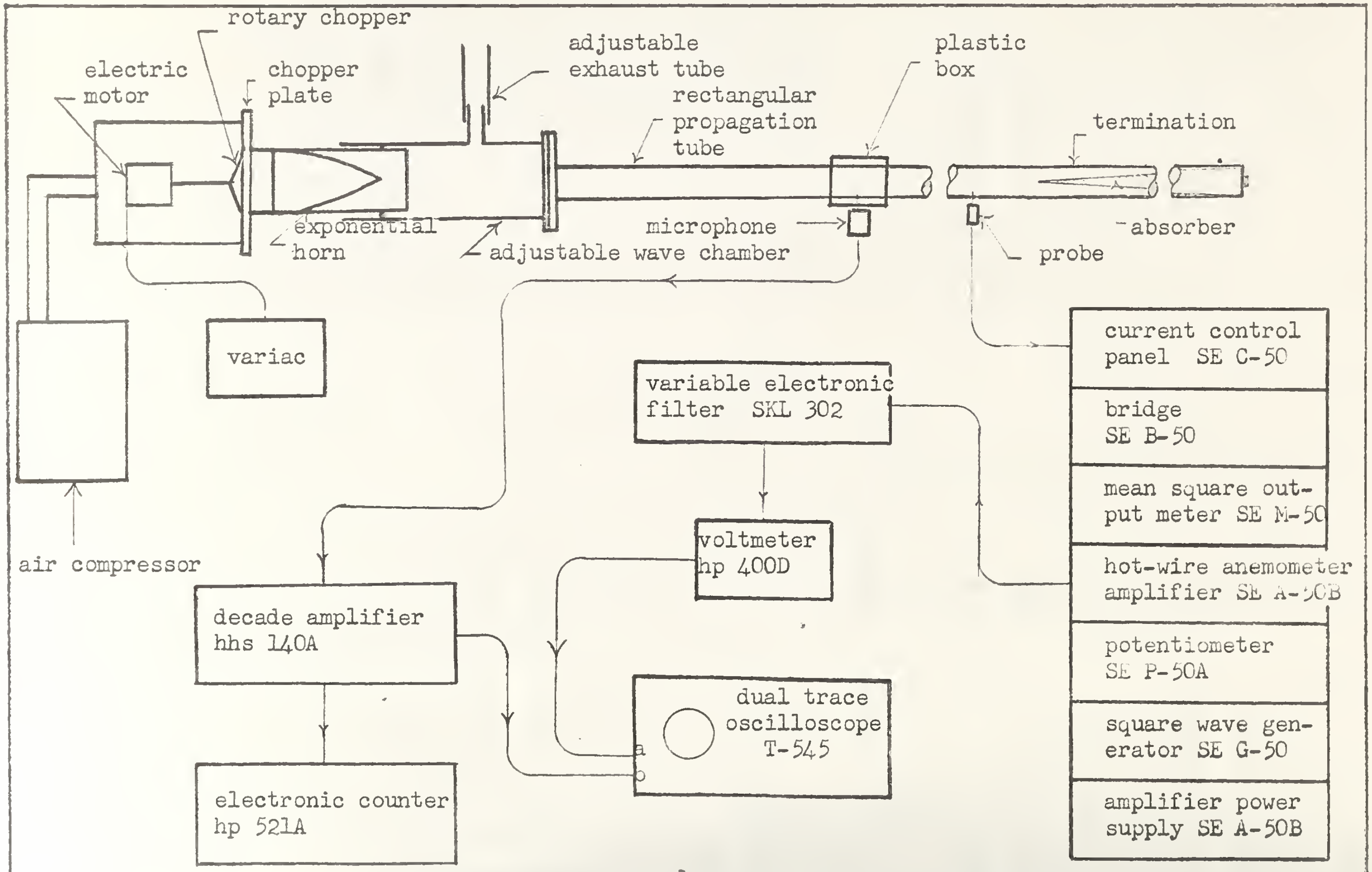


Figure 2. Schematic Diagram of Experimental Set-up

5. General Conduct of Investigation.

In section 4 of this report the following non-linear expression for determining the motion of a moving fluid by use of a hot-wire anemometer was developed:

$$i^2R_W - (T_W - T_a)(A + B'\sqrt{\rho u}\ ) = C(dT_W/dt) \qquad (6)$$

The application of this expression to the study of repeated plane shock waves presents a unique problem. The rapid succession of compression and rarefactions at a fixed point in the tube creates periodic changes in pressure, density, velocity, and temperature of the fluid medium. At "large" distances from the wall, the nature of the velocity is assumed to consist of an alternating flow which is in phase with the pressure changes, and some induced steady secondary motion (acoustic streaming) as a result of viscous forces. This complex flow field may be further complicated by the existence of turbulence.

The inability of the hot-wire to follow rapid velocity changes is reflected in equation (6) by the existence of the term on the right. Some means must be devised to determine this response, or to operate under a set of conditions in which the output intelligence of the wire can essentially reduce this term to zero. The empirical coefficients, A & B are temperature sensitive and their values must be determined over the expected temperature range (T_W - T_a). Therefore, all terms in equation (6) are variable except the current "i" which is a controlled

input and is maintained constant by the current control section of the hot-wire anemometer. The conversion of equation (6) to a more useful form is desirable:

$$V_W = (T_W - T_a)(A + B'\sqrt{\rho u})/i - (C/i)(dT_W/dt) \qquad (7)$$

where:

$$V_W = i\ R_W$$

V_W in actuality is the useful output of the hot-wire anemometer. The above expression, although valid for any fluid, is applied in this experiment to air at near standard conditions.

. Thermodynamic Relationships.

Although, strictly speaking, the relationships between thermodynamic variables during the passage of a shock front are described by the Rankine-Hugoniot equations, for the pressure swings in this experiment the relations are considered to be essentially reversible adiabatic, where:

$$\rho_1 = \rho_0 (P_1/P_0)^{\gamma} \qquad (8)$$

$$T_1 = T_0 (P_0/P_1)^{\frac{1-\gamma}{\gamma}} \qquad (9)$$

and

ρ_0 = density of air at absolute pressure P_0

ρ_1 = density of air at absolute pressure P_1

T_0 = temperature of air at absolute pressure P_0

T_1 = temperature of air at absolute pressure P_1

γ = 1.4 for air

To consider the concurrent density, velocity, and temperature variations, a typical value of $(P_1 - P_0)$ is assumed to be one-tenth atmosphere.

Variations in ρ are computed to be approximately equal to ± 4% and therefore, for the ranges which are considered in this experiment, ρ will be considered constant.

Defining $B = B' \sqrt{\rho}$

Equation (6) now becomes:

$$i^2 R_W = (T_W - T_a)(A + B\sqrt{u}) - C(dT_W/dt) \qquad (10)$$

Equation (7) now becomes:

$$V_W = (T_W - T_a)(A + B\sqrt{u})/i - (C/i)(dT_W/dt) \qquad (11)$$

During shock passage, variations in temperature T_a are approximately $\pm 9^{\circ}C$. The fluid velocity during the shock passage varies from a positive maximum of approximately 24 m/sec through zero to a negative maximum of approximately 24 m/sec.

7. Empirical coefficient evaluation.

In order to determine the effect of the shock temperature swing upon V_W, the hot-wire probe was inserted into an oven. The temperature of the air within the oven, T_a, was brought down to below its expected range by filling the oven with dry ice. The hot-wire probe and thermometer were protected from convection currents. The dry ice was removed and the inside of the oven was allowed to gradually heat up over the entire range of T_a during the shock wave passage. Under these conditions equation (10) becomes modified by setting "u" equal to zero, and $C(dT_W/dt)$ approximately equal to zero.

$$i^2 R_W = (T_W - T_a)\, A \qquad (12)$$

where:

$$R_W = R_o(1 + \alpha_o T_W)$$

and

R_o = the measured resistance of the wire at T_o

α_o = the temperature coefficient of resistance of the wire.

Values of R_W were read directly from the resistance bridge. Fig. 3 shows values of R_W plotted against $(T_W - T_a)$ for various hot-wire probes at specified wire currents, whereas Fig. 4 is a plot of R_W plotted against $(T_W - T_a)$ for a specified hot-wire probe at various wire currents. Fig. 5 shows values of A plotted against $(T_W - T_a)$. With these known values of A over the expected range of $(T_W - T_a)$,

[illegible] be performed [illegible] experiment at a constant T_w, under known velocities of steady laminar flow.

For quasi-stationary conditions, $C(dT_W/dt)$ is assumed approximately equal to zero and equation (10) now becomes:

$$i^2 R_W \doteq (T_W - T_a)(A + B\sqrt{u}) \qquad (13)$$

From this expression, values of B can be determined from known values of "u". The velocity "u" was determined by means of a pitot tube and a micromanometer. Values of B plotted against $(T_W - T_a)$ are shown in Fig. 6.

Figure 3

R_w vs. $(T_w - T_a)$

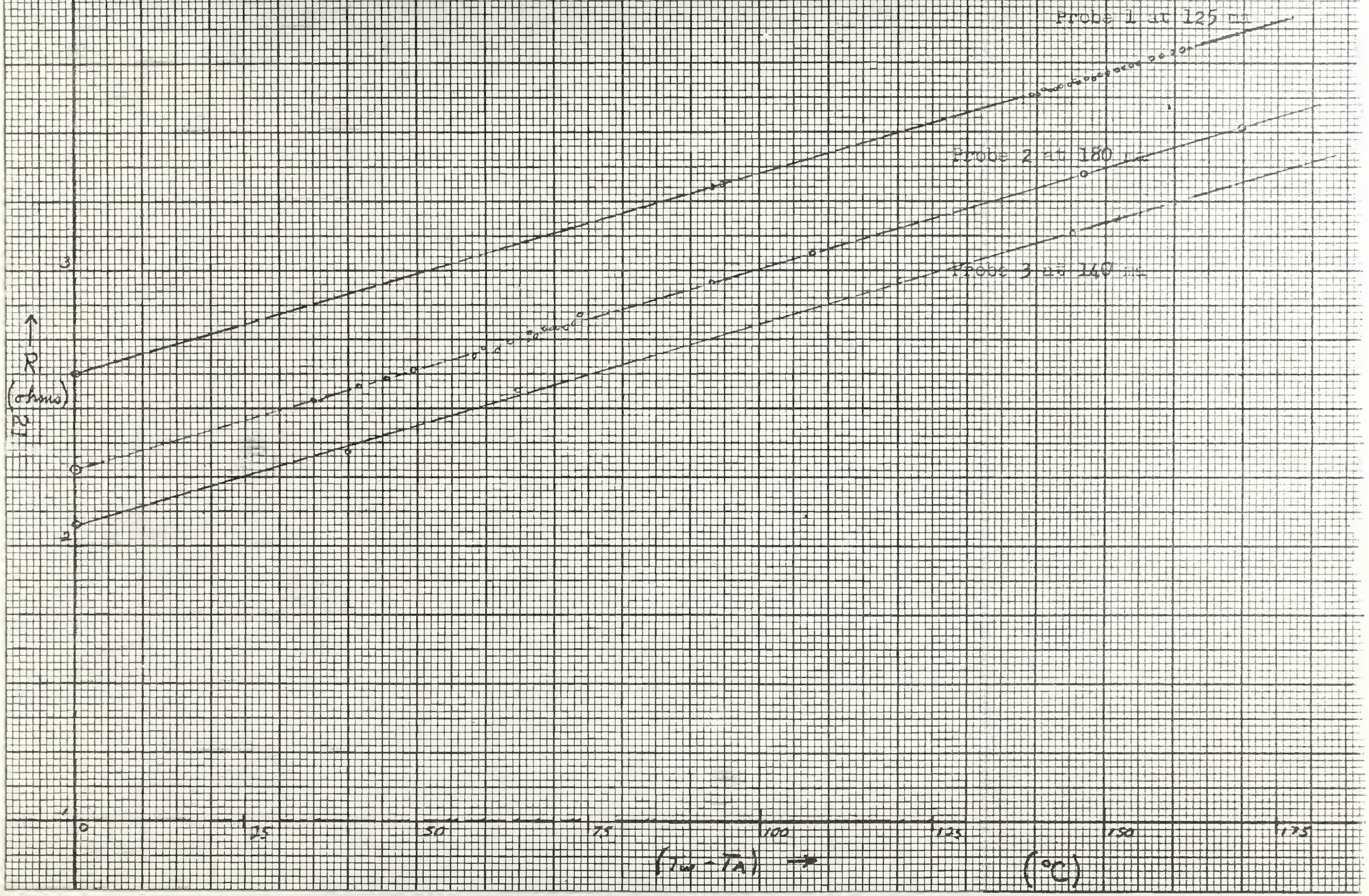
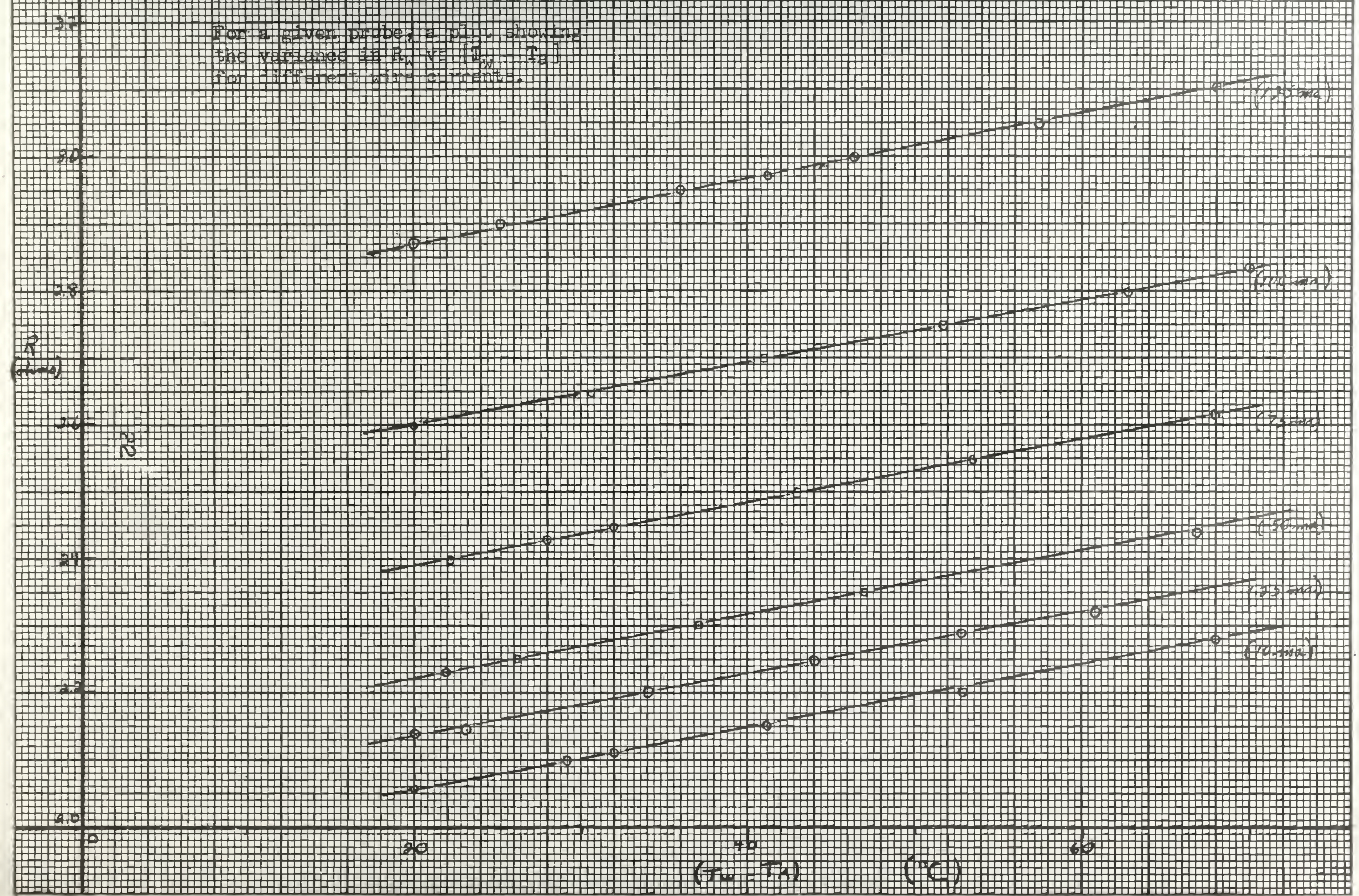
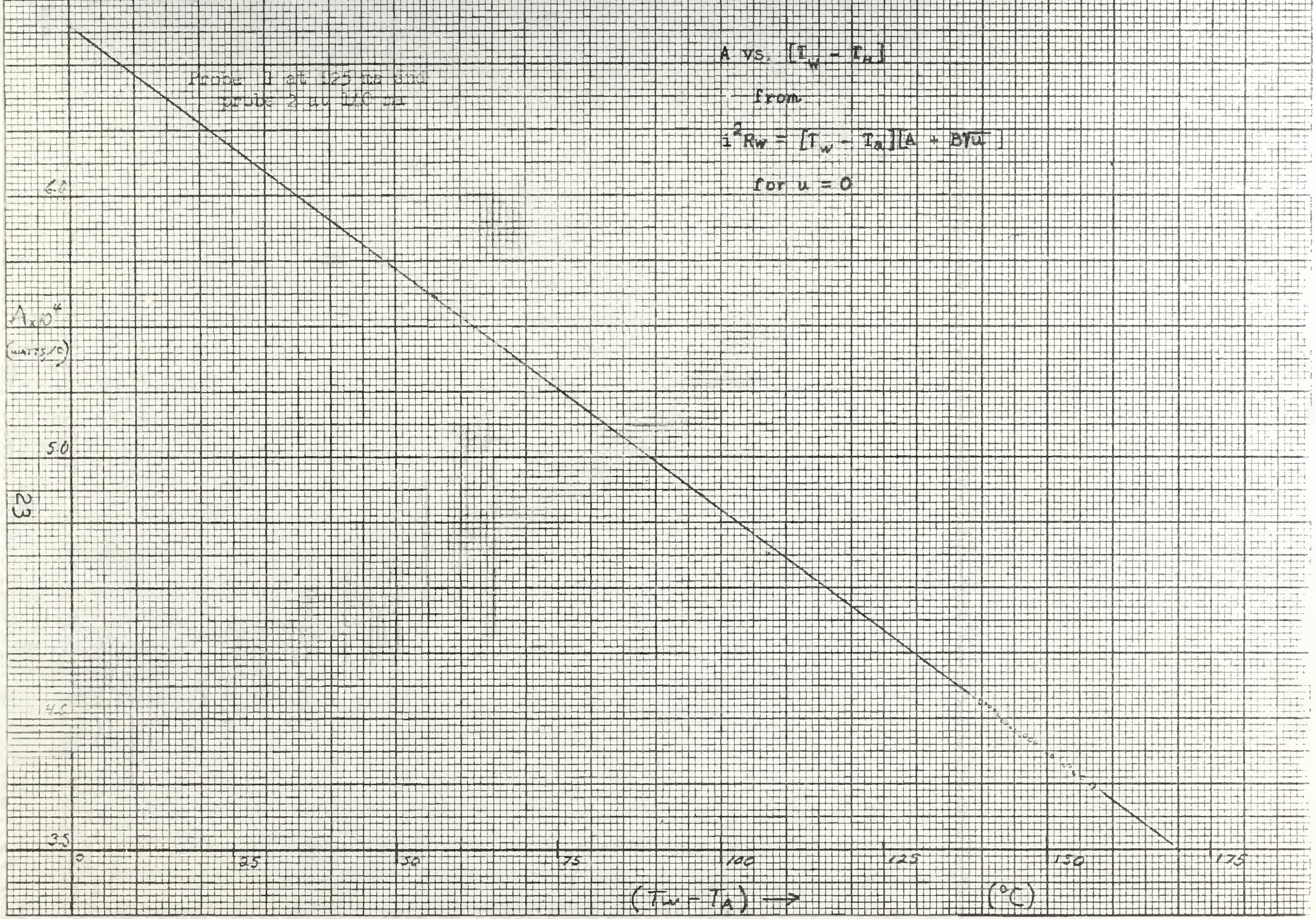


Figure 4

For a given probe, a plot showing the variance in R_w vs $(T_w - T_a)$ for different wire currents.

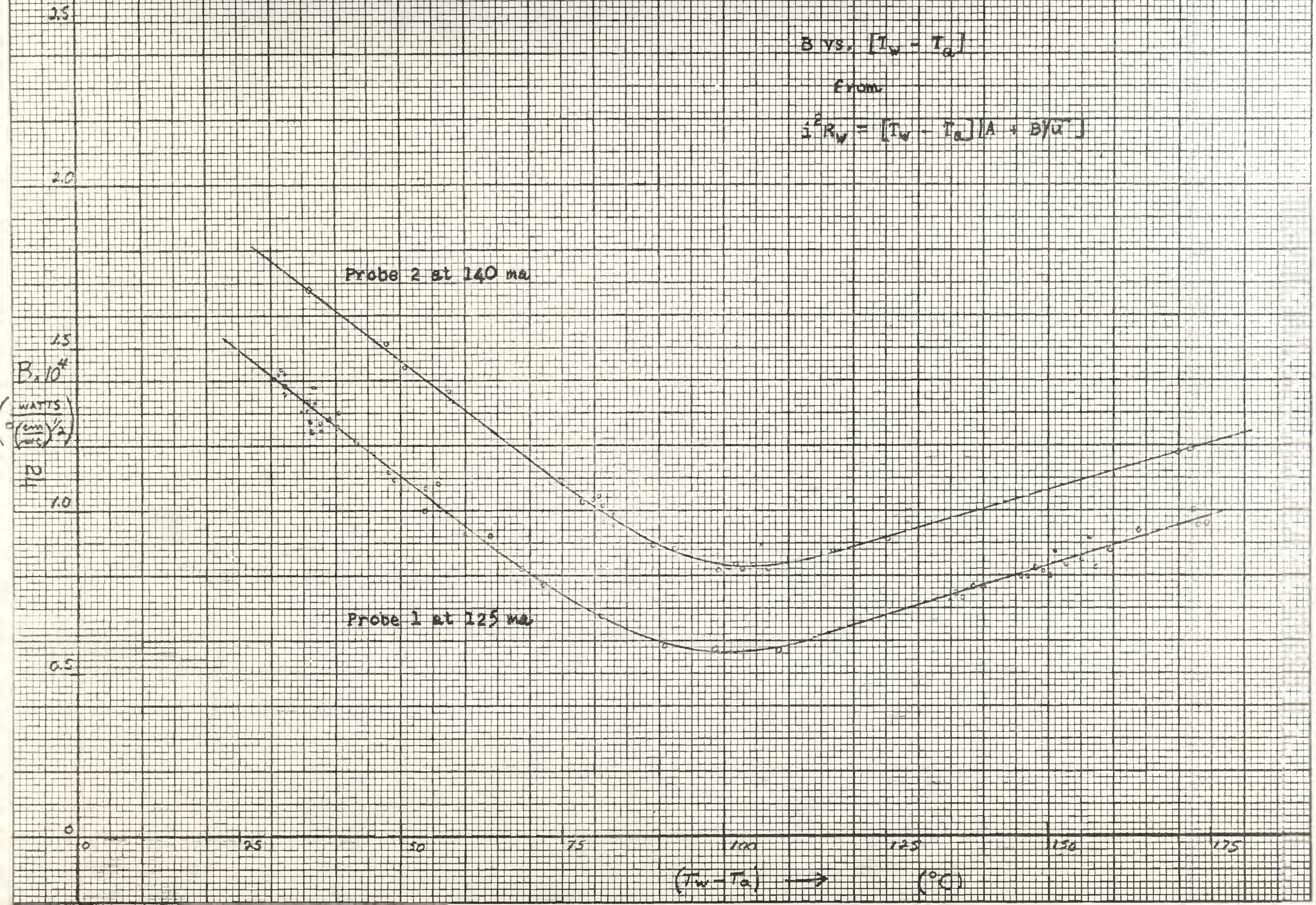




Probe 1 at 125 ma and
probe 2 at 110 ma
A vs. $[T_w - T_a]$
from
$i^2 R_w = [T_w - T_a][A + B\sqrt{u}]$
for $u = 0$
$A \times 10^4$
(watts/C)
6.0
5.0
4.0
3.5
0
25
50
75
100
125
150
175
$(T_w - T_A) \longrightarrow$
(°C)

Figure 5

Figure 6



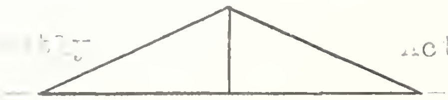
B vs. $[T_w - T_a]$
from
$i^2 R_w = [T_w - T_a][A + B\sqrt{u}]$
Probe 2 at 140 ma
Probe 1 at 125 ma
$B \times 10^4$
$\left(\frac{\text{WATTS}}{(\text{cm/sec})^{1/2}}\right)$
0
0.5
1.0
1.5
2.0
2.5
0
25
50
75
100
125
150
175
$(T_w - T_a) \longrightarrow$ (°C)

8. Qualitative Theoretical Analysis of the Complex Flow Field Near the Center of the Tube.

The final simplification of equation (10) requires consideration of the conditions which justify elimination of the last term. The assumption will be made that the velocity is in phase with the pressure swing of the shock wave on the axis of the tube. Although equation (11) shows that V_W is not a linear function of u, in order to simplify the graphs, linearity will be assumed for the discussion in this section. The effect of the actual relationship between V_W and u would be to introduce harmonic distortion and will be discussed in Section 9. Fig. 7a shows a plot of pressure against time for a repeated plane shock wave passing a fixed point. A Fourier analysis of this saw-tooth waveform shows that such a wave is composed of a fundamental and an infinite number of harmonics of amplitude inversely proportional to the number of the harmonic. Fig. 7b shows the assumed alternating velocity flow at a fixed point at the center of the tube as a result of pressure variations and in the absence of any acoustic streaming. The response of a hot-wire anemometer is independent of the direction of flow. The theoretical response of a hot-wire anemometer, which is capable of closely following these variations and producing a voltage V_W in phase with the velocity is plotted in Fig. 7c. (Actually, Fig. 7c is applicable to both V_W and $|u|$.) Particular attention

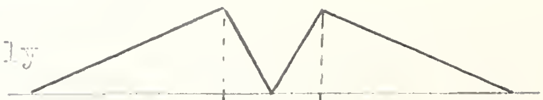
[illegible] discontinuities that exist in these waveforms at the point designated as t_o.

Apparently



t_o

Actually



t_o

In actuality, t_o is a finite period of time, a typical value for a shock of this strength being of the order of 10^{-8} seconds[1]. Within this period of time, both pressure and velocity change from peak positive values to peak negative values.[2]

It is interesting to note, Fig. 7c, that on a theoretical basis, because of the full wave rectifying action of the wire, the second harmonic of the shock wave is the lowest contained frequency in the voltage response produced by the hot-wire. This is shown in the above diagram. Two possible exceptions in which the hot-wire anemometer will produce a voltage of the <u>same</u> frequency as the frequency of the velocity fluctuations are illustrated in Figures 7d and 7e.

[1] L. Talbot and F. S. Sherman, Structure of Weak Shock Waves in a Monatomic Gas, NASA Memo 12-14-58W, pp 49, January 1959.

[2] Provisions are made in a hot-wire anemometer amplifier to electronically adjust (compensate) for the anemometer's inability to follow the high frequency components that exist in discontinuous step functions. However, the amount of compensation to be used is based on the existence of some steady state flow prior to, and after the discontinuity. These conditions are not present for a sawtooth wave. Furthermore, hot-wire anemometer time constant adjustments which permit the high frequency components to be more efficiently represented, cause a relative decrease in the low frequency response, which is dependent on the mean flow conditions.

In Fig. 7d, a linear hot-wire response to alternating flow would show the inherent time lag in the wire due to the wire's inability to follow high frequency changes. Because of this lag, there is a general rounding off from the severe response illustrated in Fig. 7c. There is a slight shift in phase between u and V_w and the duration t_o is ignored by the wire, thereby producing a signal containing the fundamental frequency of the shock wave.

Fig. 7e shows a linear theoretical response of a hot-wire anemometer to an alternating velocity, superimposed on acoustic streaming of a larger absolute value than the alternating component. Again the anemometer would produce a signal containing the fundamental frequency.

Recognizing that DC flow, with an expected magnitude equal to $0.54\ u^2/c_o$ [9], could affect the fundamental frequency, attempts were made to physically separate the alternating flow from the acoustic streaming by isolating a small section of the propagation tube with diaphragms. The technique for accomplishing this was to use a diaphragm material which was thin and pliable enough to transmit the discontinuous pressure pulse, and to restrict the length of the isolated section between diaphragms to prevent regeneration of induced acoustic streaming.

In general, this experiment failed to produce its objective because no suitable diaphragm material was found that was strong enough to withstand the shock passage and

resemble a fully developed shock wave. Neoprene rubber, one-eighth inch thick proved to be the most durable material, withstanding the vibrational abuse up to periods of approximately five minutes; however, its attenuating effect on the pressure pulse was too great. Plastic mylar and latex rubber of various thicknesses immediately burst upon impact of the pressure pulse. The most satisfactory material was dental rubber (rubber dam) which permitted operating periods of approximately one minute. In spite of this, due to the fact that there was not sufficient distance in the section for build up, the characteristic sawtooth waveform was not quite achieved.

The isolation of the alternating flow from the acoustic streaming is probably impossible with any materials available at this time.

Not being able to control the nature of the flow in the tube, the only alternative was to analyze it. Figures 8a, 8b, and 8c are oscillograms of uncompensated hot-wire anemometer response together with the voltage response of the barium titanate transducer (these are dual trace oscilloscope photographs, the lower trace representing the alternating pressure of the shock wave.) The resemblance of Fig. 8 to Fig. 7d verifies the existence of the fundamental frequency in the response of the hot-wire anemometer and vividly points out the anemometer's inability to detect the events occuring during the time

[illegible] components [illegible] of [illegible] wave.

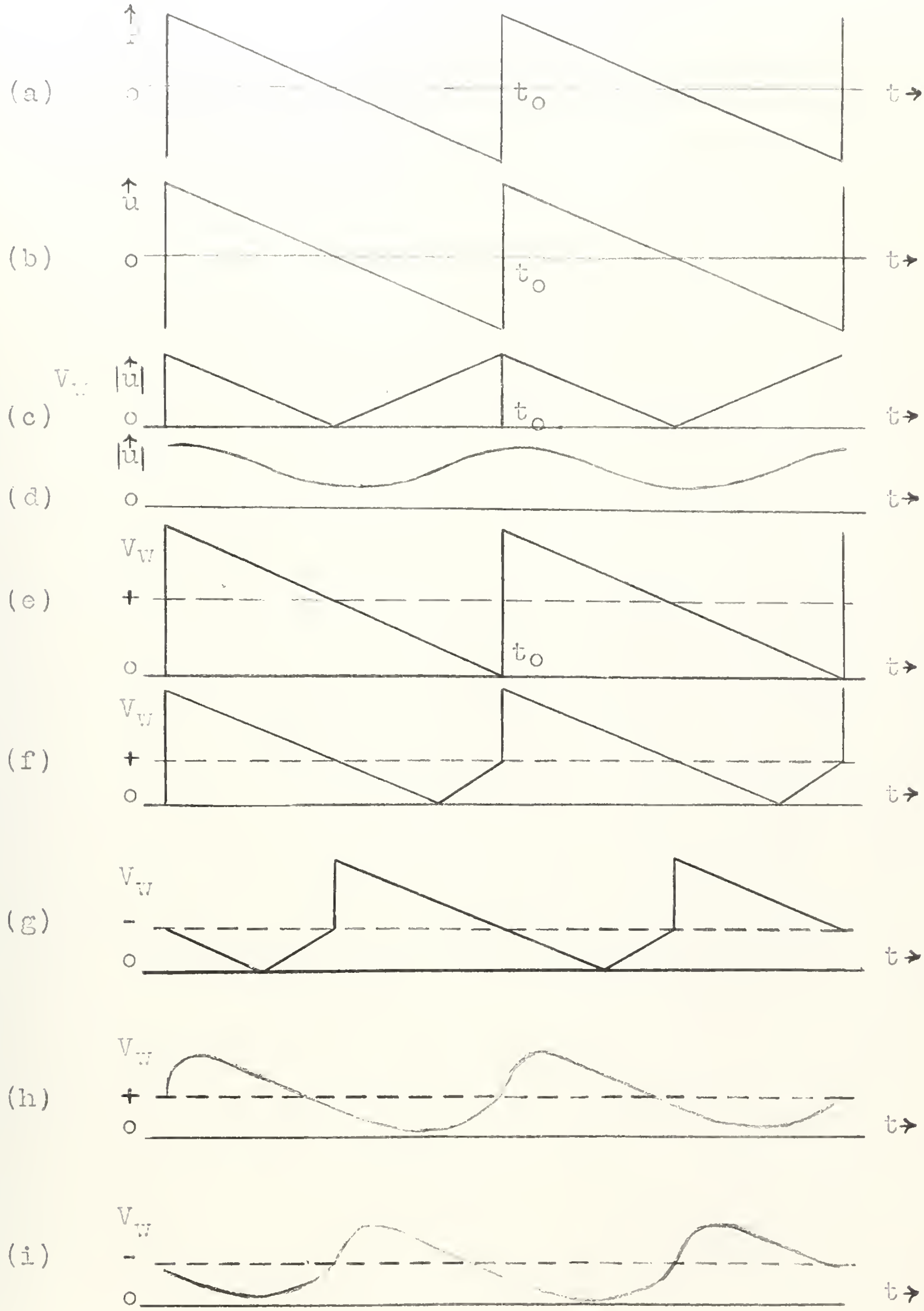
Fig. 7f is a theoretical linear response of the hot-wire anemometer for acoustic streaming of amplitude one-half that of the first harmonic. This assumed streaming is probably a maximum value, (most likely it is four to five times the actual value) but its usage points out the maximum effect that streaming could have. Since the end of the propagation tube was plugged, the principle of continuity demands that an equal and opposite flow takes place somewhere in the cross-section of the tube. Analytically, $\int \rho \, u dA = 0$.

Fig. 7g is a theoretical linear response of the hot-wire anemometer for acoustic streaming, equal to that in Fig. 7f in magnitude, but opposite in direction.

Figures 7h and 7i are estimates of what the actual uncompensated, yet linear, hot-wire anemometer response with time lag would be for the situations described in Figures 7f and 7g respectively.

What is of fundamental importance here is that if well defined acoustic streaming of a significant amount exists, it may be detected by the following criteria:

a. If the magnitude of the acoustic streaming is insignificant, then the characteristic response of the hot-wire anemometer contains the fundamental frequency of

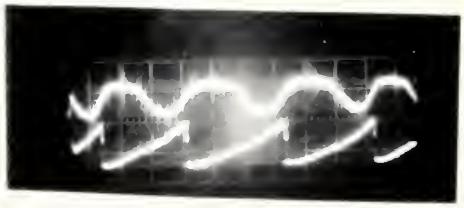


Theoretical Hot-Wire Anemometer Response Curves

Figure 7

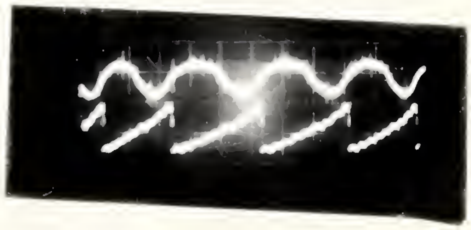
a. 365 cps.

3 psi.



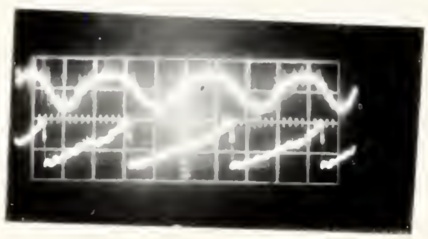
b. 600 cps.

3 psi.



c. 895 cps.

3 psi.



Typical Hot-Wire Anemometer
and Barium Titanate Transducer
Response Oscillograms

Figure 8

the shock wave, the wave form is symmetrical, and no phase change is detected as the hot-wire anemometer probe traverses the tube.

b. If the magnitude of the acoustic streaming is detectable, but less than the magnitude of the alternating velocity, the characteristic response of the hot-wire anemometer contains the fundamental frequency of the shock wave, the wave is asymmetrical and a phase change will be detected as the probe traverses across the tube.

Proceeding on the above assumed criteria for determining the relative amount of acoustic streaming in the propagation tube, the hot-wire anemometer was moved across the tube. No detectable phase changes were noted. Furthermore, the typical response curves for the hot-wire anemometer, as shown in Fig. 8, show almost perfect symmetry except for a noticeable concave distortion in the trailing edge of the response wave. On the basis of this evidence the following assumptions are made:

a. The ratio of the magnitude of acoustic streaming to alternating velocity is negligible.

b. The inability of the hot-wire anemometer to respond to the high frequency components of the sawtooth waveform hinders its utilization for determining absolute velocity.

c. The existence of the fundamental frequency in the output of the hot-wire anemometer provides a means of

utilizing this low frequency in the analysis of the sawtooth waveform.

Verification for assumption c., immediately above, was gained by observing cigarette smoke through the plexiglass sections. This qualitative experiment showed no noticeable acoustic streaming, however, low frequency turbulence was observed.

A criterion for the existence of turbulence in DC flow is given by the following expression (8):

$$R_{crit} = (\bar{u}d/\nu)_{crit} \cong 2300 \tag{14}$$

where:

R_{crit} = lower limit of Reynold's number for a pipe, below which the flow does not become turbulent.

$\bar{u}$ = the mean velocity averaged over the cross sectional area of the pipe.

d = effective diameter of the pipe.

ν = kinematic viscosity..

Substituting the following values in equation (14):

R_{crit} = 2300

d = 0.05 meters

ν = 15.55×10^{-6} meters^2/sec

The resulting evaluation for $\bar{u}$ indicates that turbulent flow will result for $\bar{u} \geq 0.72$ meters/sec. To provide a comparison between this value of acoustic streaming with the peak alternating velocities, it is necessary to evaluate the Rankine-Hugoniot relationship for particle

velocity:

$$u_2 = a_x \left[\frac{(2/\gamma)(P_y/P_x - 1)^2}{(\gamma + 1)(P_y/P_x) + (\gamma - 1)} \right] \tag{15}$$

where:

a_x = ambient speed of sound

γ = the ratio of specific heats for air

P_x = ambient pressure

P_y = pressure after shock passage

Substituting the following values in equation (15):

a_x = 350 meters/sec

γ = 1.4 for air

P_y/P_x = 1.1

an evaluation for u_2 indicates that peak velocities of 24 meters/sec are to be expected. From these calculations and the observations of smoke particle behavior and wave shape stability, it is assumed that the ratio of acoustic streaming to alternating velocity is negligible.

Assumption b. above, again points out that unless some further means is devised for analyzing the hot-wire anemometer output, the quantity $C(dT_w/dt)$ is an undetermined quantity.

Assumption c. above offers such a means for analysis. Figure (9) is a plot of frequency response vs. frequency for the hot-wire anemometer probe utilized for this experiment. It indicates that frequency response is good over the indicated range of 300 to 900 cps, which is the range of the fundamental frequency of the shock wave. The

peak amplitude of a sawtooth wave may be expressed by the following:

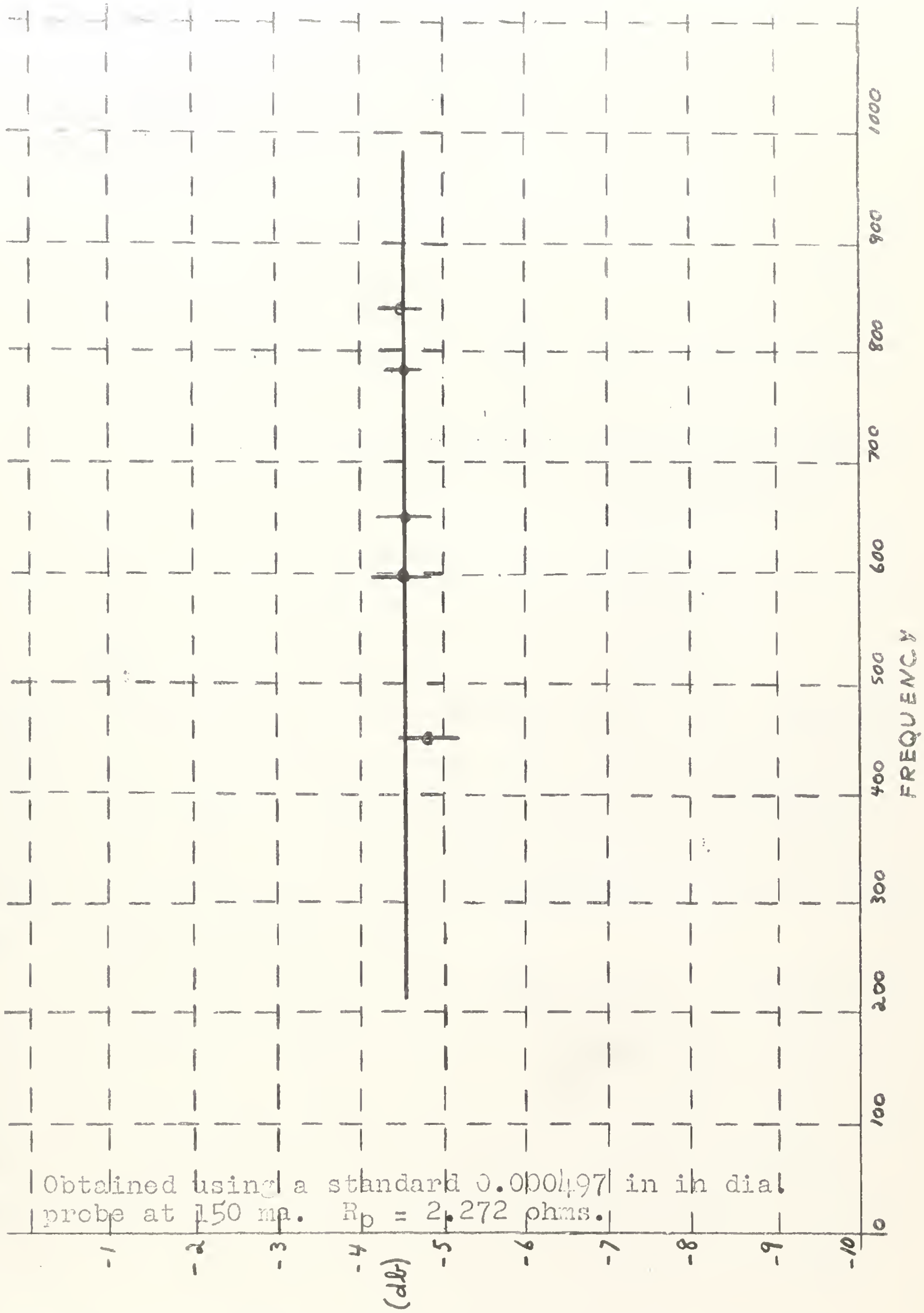
$$P = (2P_0/\pi)(\sin\omega t - \tfrac{1}{2}\sin 2\omega t + \tfrac{1}{3}\sin 3\omega t - \tfrac{1}{4}\sin 4\omega t \cdot \cdot \cdot + \tfrac{1}{n}\sin n\omega t) \qquad (16)$$

From this expression it can be seen that the fundamental frequency contributes $2/\pi$ to the peak value.

Hot-wire anemometer filaments are available which will easily follow frequencies of the order of 300 to 1000 cps. Electronic filtering of the hot-wire output will allow only the fundamental to contribute to a signal which when displayed on an rms voltmeter could be related to the peak value by the following expression:

$$V_{peak} = \sqrt{2}\,\pi V_{rms}/2 \qquad (17)$$

Under these conditions, where the filament utilized is capable of following the fundamental, $C(dT_w/dt)$ can be considered approximately equal to zero and equation (10) may therefore be simplified to equation (13).



Fundamental Frequency Response of Hot-Wire Anemometer

Figure 9

F. Non-Linear Corrections to Theoretical Analysis

The qualitative statements of the previous section were based on the assumption of a linear curve for V_W vs. u. For actual wire response, the behavior is far more complicated. Using equation (12):

$$V_W = (T_W - T_a)(A + B\sqrt{u})/i \qquad (12)$$

and utilizing values of A and B for a chosen $(T_W - T_a)$ of 165°C,

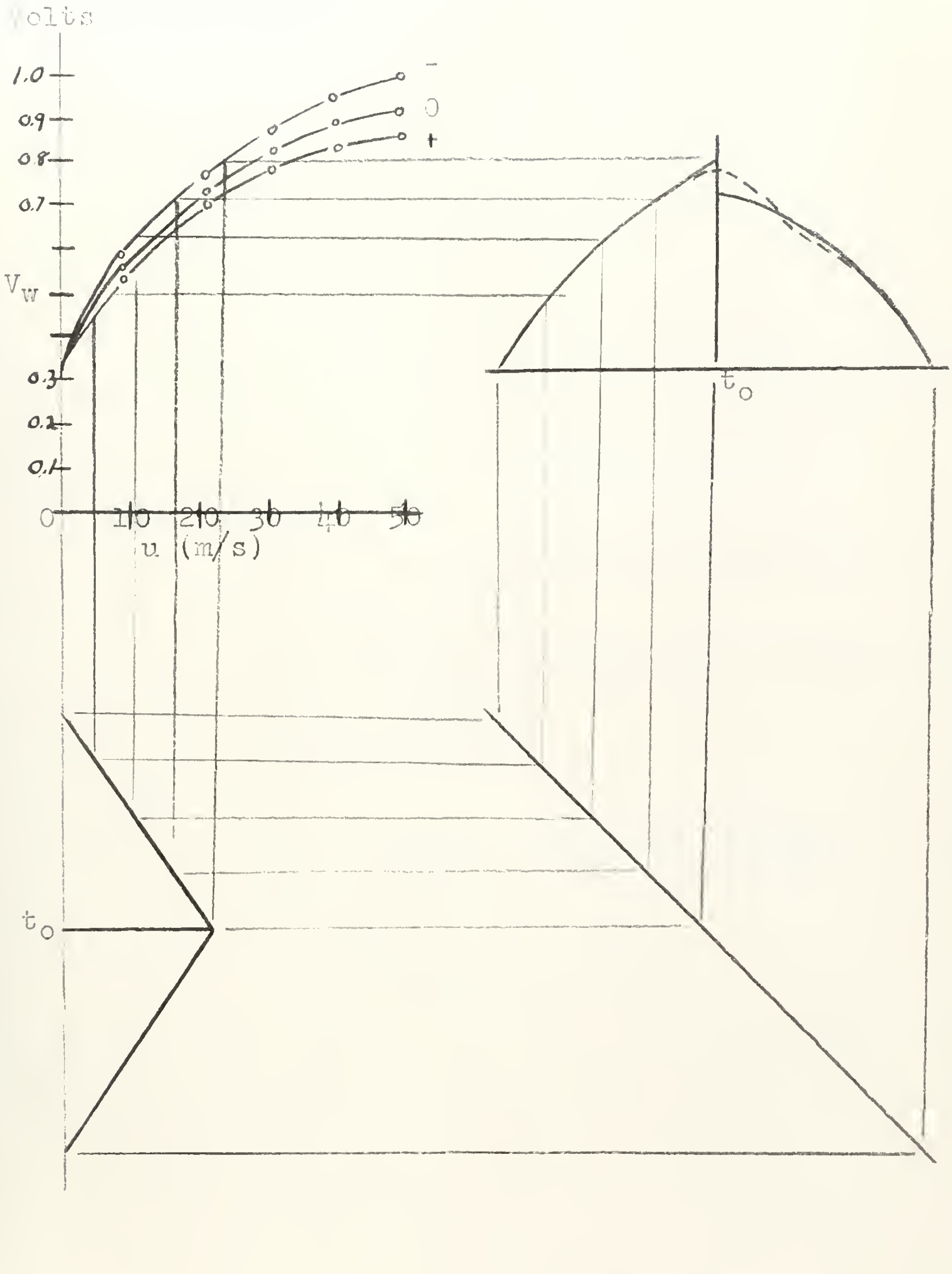
$(T_W - T_a)$	A (10^4 watts/°C)	B $\left(\dfrac{10^4 \text{ watts sec}^{\frac{1}{2}}}{°C \text{ meters}^{\frac{1}{2}}}\right)$
156	3.76	0.87
165	3.60	0.93
174	3.42	0.99

a graph of equation (12) is plotted as Fig. 10. The wire current was maintained at 175 ma and R_o was 2.272 ohms. An upper and a lower limit of $(T_W - T_a)$ is included as a result of the reversible adiabatic changes in T_a of 9°C from the ambient temperature.

The wave form of Fig. 7c is assumed and a graphical "characteristics" method is used to predict the output. For a constant $(T_W - T_a)$, the non-linear effect would change the waveshape but would not change the symmetry. However, since $(T_W - T_a)$ is not a constant during shock passage, it is incorrect to utilize one curve of King's law for the non-linearity corrections during the entire velocity cycle.

A more precise, but still approximate, method would be to utilize the middle curve (ambient temperature) for the velocities near zero, the upper curve (ambient minus 9°C) for the peak velocities corresponding to the rarefaction part of the cycle, and the lower curve (ambient plus 9°C) corresponding to velocities during the compression part of the cycle, with intersolution for intermediate values. This method introduces asymmetry into the resulting waveform which is a result of the wire transmitting a continuous representation of the discontinuous temperature change at t_o and illustrated by the dotted line in Fig. 10. This appears to account for the concave appearance of the typical oscillograms shown in Fig. 8.

On the basis of these results, it seems that the effects of changes in T_a, are significant, particularly for smaller values of $(T_w - T_a)$.



Non-Linear Correction Curve for a $(T_W - T_a)$ of 165°C

Figure 10

10. Boundary Layer Observations.

So far, the entire preliminary investigation into the possible application of hot-wire anemometry to the study of particle velocities in repeated plane shock waves has been limited to the conditions that exist near the center of the tube. The final phase of this study was to determine the effect of the tube wall on velocity. Figures 11, 12, and 13 show how the rms voltage of the harmonic components varies with distance from the tube wall. They are plotted in the dimensionless units $(E_y/E_{\alpha})^2$ vs. η, where:

E_{α} = the rms voltage at the center of the tube

E_y = the rms voltage at a distance y from the wall of the tube.

$\eta = y\sqrt{\omega/2\nu}$

and

y = distance from wall of tube

ω = frequency

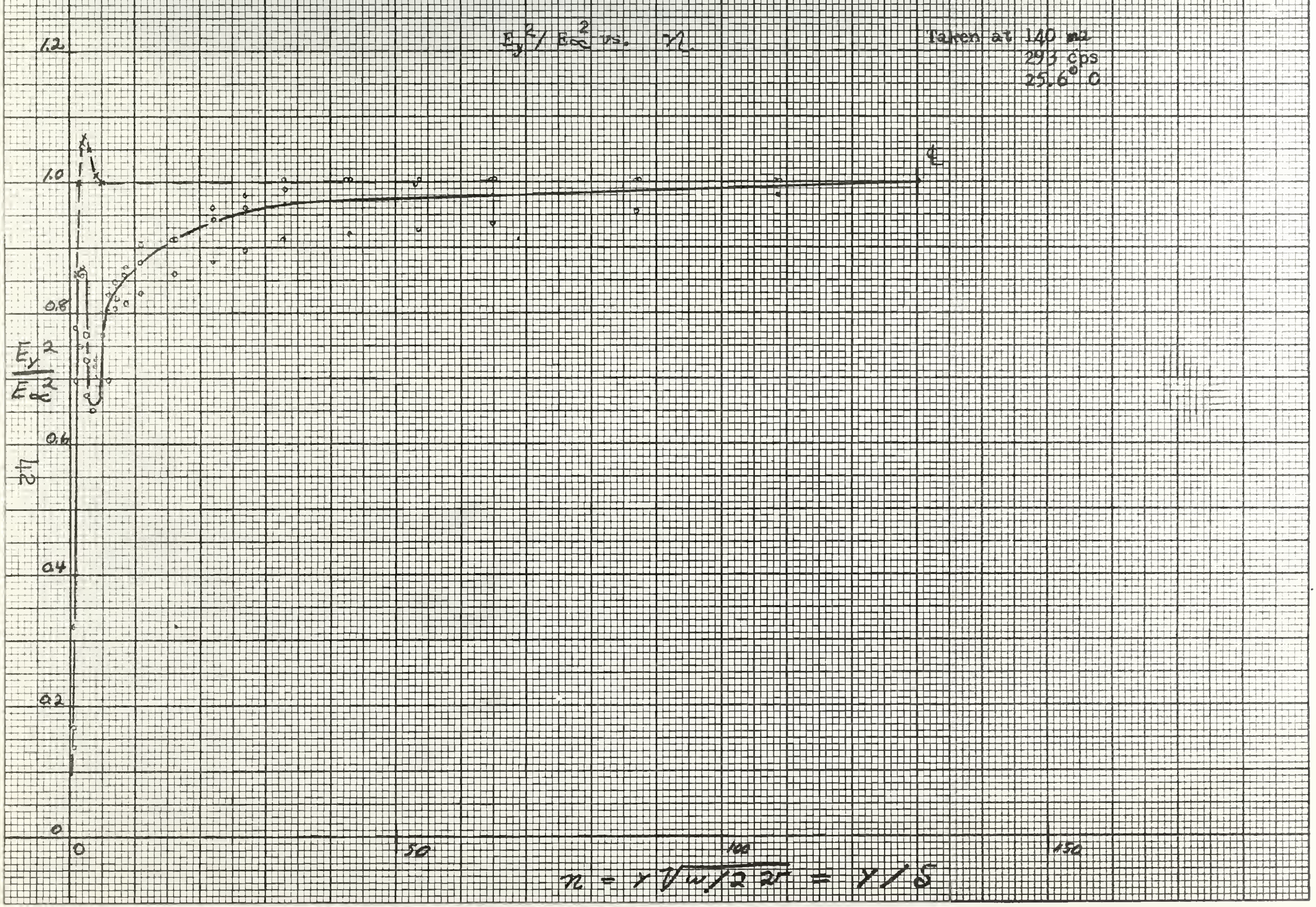
ν = kinematic viscosity

A dashed line, superimposed on the above mentioned curves shows the shear profile as determined by Richardson (8).

During the experiment, electronic filtering was used so that the fundamental frequency was the primary source of information. Under the assumption of the preceding development, this is considered to be the most reliable source of information. These plots show that near the

wall of the tube, increases in particle velocity are observed. These increases vary in magnitude and location with changes in frequency. No explanation is offered at this time for the occurence of this phenomenon. Figure 13 shows in addition to the fundamental frequency of 800 cps, the results of electronically filtering out all but the second harmonic of 1600 cps. The fact that the velocities of these two frequencies are maximized at the same distances from the tube wall may indicate that the shock wave structure is unchanged at close proximity to the wall.

Figure 11



Ey²/E∞² vs. η
Taken at 140 ma
293 cps
25.6° C
1.2
1.0
0.8
0.6
0.4
0.2
0
Ey²/E∞²
0
50
100
150
η = y √ω/2ν = y/δ

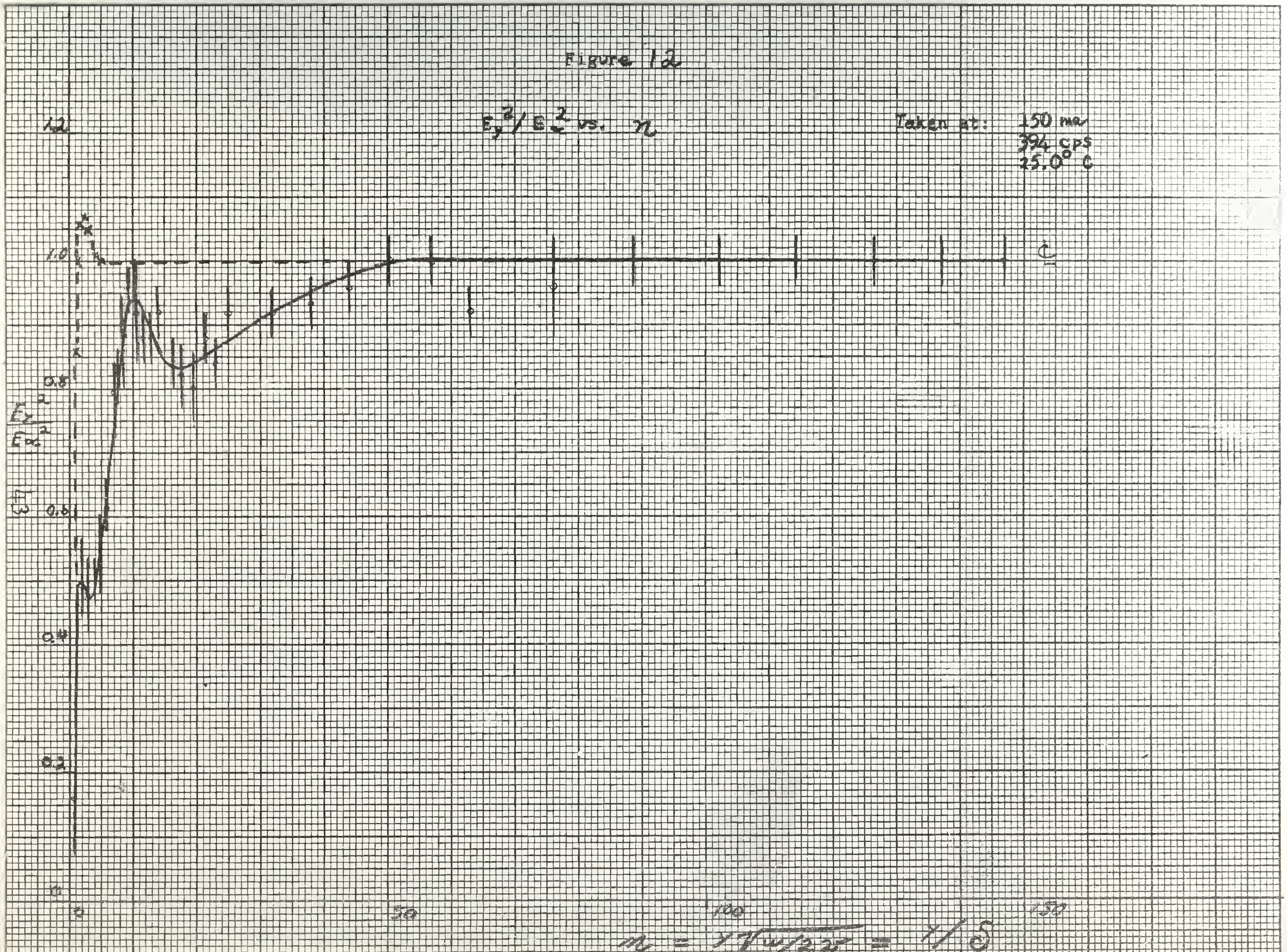


Figure 12
E_y^2 / E_∞^2 vs. η
Taken at: 150 ma
394 cps
25.0° C
1.2
1.0
0.8
0.6
0.4
0.2
0
$\frac{E_y^2}{E_\infty^2}$
50
100
150
$\eta = y\sqrt{\omega/2\nu} = y/\delta$

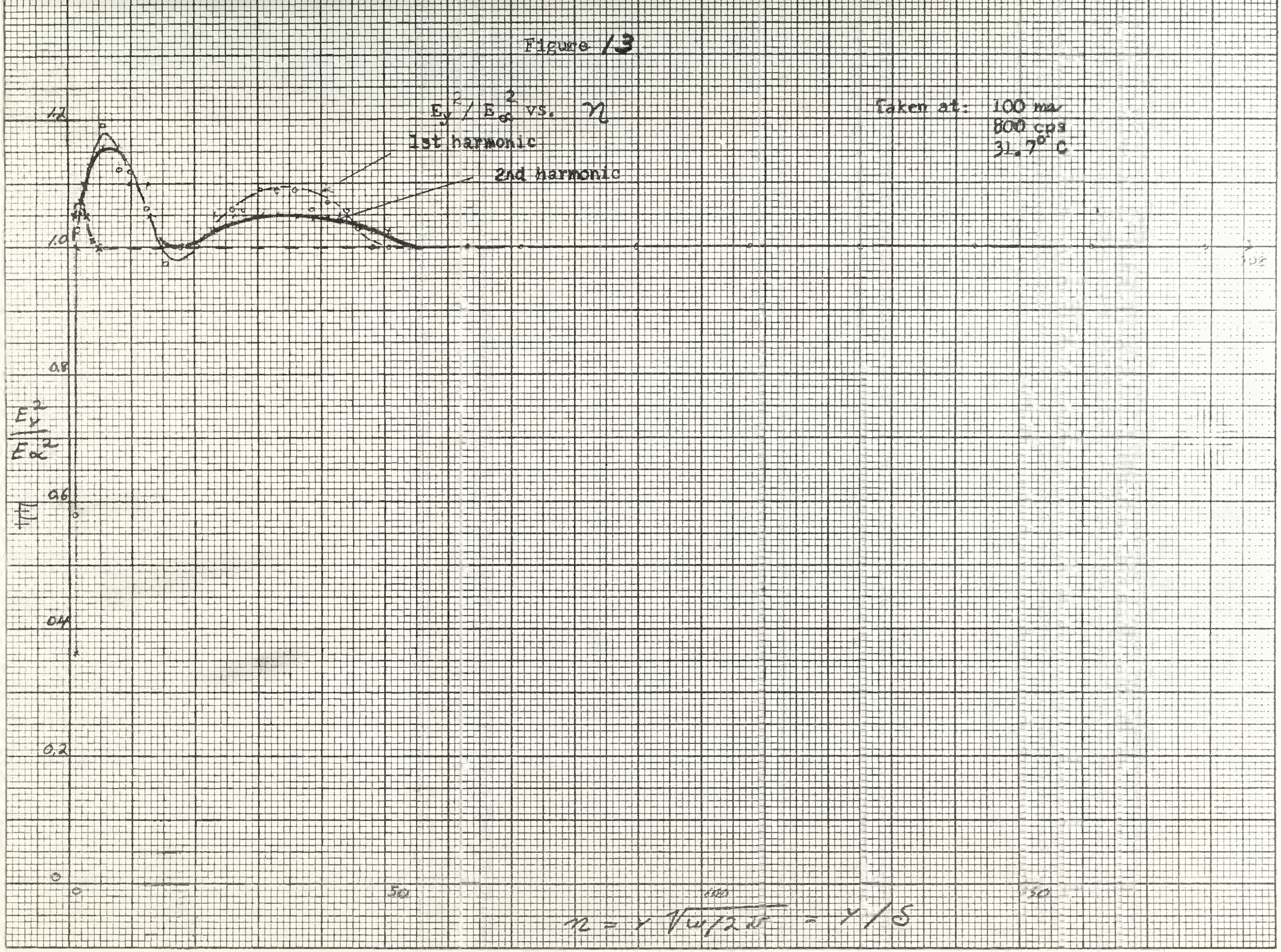


Figure 13
E_y^2 / E_α^2 vs. η
1st harmonic
2nd harmonic
Taken at: 100 ma
800 cps
31.7° C
1.2
1.0
0.8
0.6
0.4
0.2
0
$\frac{E_y^2}{E_\alpha^2}$
0
50
100
150
$\eta = y\sqrt{\omega/2\nu} = y/\delta$

11. Conclusions.

This experiment has revealed a considerable amount of information concerning the nature of the complex flow field associated with the passage of repeated plane shock waves in a shock tube, particularly near the wall of the tube. Acoustic streaming is present, but apparently is turbulent rather than laminar.

The effect of streaming upon the far greater alternating velocities appears negligible, and may be neglected for the purposes of future study of this phenomenon by hot-wire anemometry.

Although the hot-wire anemometer is not capable of following the high frequency components of the sawtooth wave, a quantitative knowledge of peak alternating velocities may be arrived at by measurements of the fundamental frequency, and the use of King's equation.

12. Recommendations.

The most serious limitation that prevented a more quantitative knowledge of the flow field at this time was the lack of fine frequency control and stabilization of the shock wave repetition rate. It was observed that frequency changes in the order of 1% caused variations in shock strength as great as 25%.

It is felt that possibly due to variations of the power input to the compressor, that the compressor output is not constant, but rather varies from the set output pressure by ±0.1 psi. This in turn would cause a varying force on the chopper, thus varying its speed. Correction of this difficulty could lead to the determination of whether or not the voltage input to the chopper motor is steady enough to cause it to maintain a definite rpm for a given compressor output pressure. Stabilization of compressor output pressure and chopper speed is an extremely important requirement for regulating the shock wave frequency.

Experimentation was conducted mainly with a probe filament of 0.000497 inch in diameter; a reduction in wire diameter however, would allow for greater frequency response. It was noted that T_W's above 200°C could cause a localized melting of the solder holding the filament on the probe tips; this in turn caused a change in the effective length of the filament, and thus in the resistance

of the [illegible]. [illegible], having a [illegible] in the range of 300-400°C, would thus do a better job than the thicker wire with solder of lower melting point. As stated earlier, the effect of the reversible adiabatic change in T_a can be minimized as $(T_w - T_a)$ increases. As T_w is related to R_w by the equation:

$$R_w = R_o(1 + \alpha_o T_w) \qquad (18)$$

and as R_w is also inversely proportional to the cross-sectional area of the wire, any reduction in the diameter of the probe filament will cause an increase in the T_w. This will not only minimize the effect of the variance in T_a, but will also give improved frequency response. If no current is placed across the probe filament except under those conditions when there is a flow across the probe, it is felt that a filament with a diameter of 0.0001 inch would give more favorable results.

In addition, Figures 5 and 6 reveal that A decreases as $(T_w - T_a)$ increases, whereas, B increases as $(T_w - T_a)$ increases beyond $(T_w - T_a)$ of approximately 100°C. At higher temperatures, of say the order of $(T_w - T_a)$ equal to 300°C, it can be seen that A becomes extremely small, and B, while increasing very slowly, is far greater than A. A is thus extremely small as compared to $B\sqrt{u}$, and can thus be neglected. During shock passage at high values of $(T_w - T_a)$, in spite of the reversible adiabatic

c[illegible], the [illegible] for [illegible], the value of [illegible] be considered [illegible]. [illegible] (11) may now be further simplified to:

$$i^2R_w = (T_w - T_a)\ [illegible]_o\sqrt{[illegible]} \qquad (19)$$

Thus, not only will using higher values of $(T_w - T_a)$ eliminate the effect of reversible adiabatic change in T_a, but will also cause additional simplification of the basic equation. For these reasons, it is highly recommended that, within wire limitations, the highest possible range of $(T_w - T_a)$ be used in future research.

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